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THE FLETTNER ROTOR SHIP IN THE LIGHT OF THE KUTTA-JOUKOWSKI THEORY AND OF EXPERIMENTAL RESULTS

By Frank Rizzo
Langley Memorial Aeronautical Laboratory

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Summary

In this paper the fundamental principles of the Flettner rotor ship (Reference 1) are discussed in the light of the Kutta-Joukowski theory and available experimental information on the subject.

A brief exposition of the Kutta-Joukowski theory is given and the speed of the rotor ship Buckau computed, first by using effective propulsive force obtained by the above theory, and then by direct application of wind tunnel data.

The calculation shows that, although there is a certain relation between theoretical and experimental speeds, those obtained by wind tunnel data are undoubtedly closer to the actual speeds of the ship than are those obtained by the use of theoretical propulsive force.

Introduction

In simple terms the theorem herein employed is only a special case of the Kutta-Joukowski (References 2 and 3 respectively) theory and it states that, the force normal to the fluid direction upon a length L of a cylinder of infinite length set across a rectilinear flow is equal to the product of the fluid density W/g, by the length L, by the circulation Γ around the cylinder, and by the velocity of the flow V_o, in which the cylinder is held (Reference 3).

The possibility of using rotating cylinders in aircraft has been discussed by various aeronautical publications (Reference 4); from an experimental point of view, however, as far as known, the problem has been so far undertaken only at the Rijk's Studiedienst Luchtvaart, Amsterdam, by Dr. Wolff (Reference 5) and at the wind tunnel of the National Advisory Committe for Aeronautics, by E. G. Reid (Reference 6).

The Magnus phenomenon on which the Kutta-Joukowski theory is based, has been known for some time, for Magnus announced it first in 1853 in conjunction with his ballistic experiments, and Lord Rayleigh (Reference 7) later proved mathematically that a lift is generated on a cylinder if a circular motion is superposed on a potential flow about the cylinder.

The Kutta-Joukowski Theory

Before proceeding with the application and discussion of results, the following brief exposition is given, and in the bibliography a list of references is given for the reader who wishes to go further into this subject.

In both hydrodynamics and aerodynamics there are two methods of treating the motion of a body in a fluid having given the motion and the form of the body itself. One method consists in determining the velocity potential and stream function from the condition that the contour of the body itself must be a streamline, since the fluid can only pass tangentially to the surface without penetrating it. In hydrodynamics this method of treatment is called the direct method.

In contrast to this there is the indirect method which derives its name from the fact that by mathematical trial the form of fluid motion and the shape of the body which could give rise to it may be determined. As a result of this, any streamline can then be taken as the contour of the body moving in the fluid. The latter method is the simpler of the two because it depends on differentiation, whereas the former depends on integration. Every student of naval architecture is familiar with the indirect method and its broad application to that science. Rankine (Reference 8) used combinations of sources and sinks to produce figures suitable for ship waterlines, Taylor (Reference 9) made use of a continuous source to a continuous sink combined with a uniform flow to obtain ship lines of least resistance, and Fuhrmann by application of the same principle to a three-dimensional flow extended the theory to hulls of airships (Reference 10).

By the indirect method, it has been found (Reference 11) that the motion of a rotating circular cylinder, immersed in a perfect fluid having a velocity $-V_0$ at infinity, with its axis parallel to the horizontal XY plane of a two-dimensional flow, and its transverse section parallel to the vertical XZ plane, can be represented by a complex function of the form:

$$F = -V_0 Z - \frac{K}{Z} + \frac{K'i}{\pi} \log \frac{Z}{r_0} \qquad (1)$$

where $Z = x + iz = r\,(\cos\theta + i\,\sin\theta\,) = re^{i\theta}$ (2)

and $r = \sqrt{x^2 + z^2}$

The real part of equation (1) gives the velocity potential:

$$\Phi = -V_0\,x - \frac{Kx}{r^2} - \frac{K'\theta}{\pi} \qquad (3)$$

and the imaginary part the stream function:

$$\Psi = -\,V_0\,z + \frac{Kz}{r^2} + \frac{K'}{\pi} \log \frac{r}{r_0} \qquad (4)$$

The first and second right members in the last two equations represent a flow around a circular cylinder (Fig. 1), while the third term in each stands for concentric streamlines and radial equipotential lines like those in Fig. 2. The resultant motion, from the combination of these two, is that represented by Fig. 3, as will be explained below.

Since Laplace's equation of constant density must be satisfied

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \qquad (5)$$

The velocity along the X axis at an infinite distance is:

$$\left[\frac{\partial \Phi}{\partial x}\right]_{r=\infty} = - V_o \left(1 + \frac{K}{\infty^2}\right) = - V_o \tag{6}$$

and along the Z axis:

$$\left[\frac{\partial \Phi}{\partial z}\right]_{r=\infty} = 0 \tag{7}$$

that is, a uniform flow with velocity $- V_o$ at infinity, parallel to the X axis. By using polar coordinates equation (3) becomes:

$$\Phi = - V_o \, r \cos\theta - \frac{K}{r^2} \, r \cos\theta - \frac{K'\theta}{\pi}$$

from which the radial velocity is

$$V_r = \frac{\partial \Phi}{\partial r} = - V_o \cos\theta + \frac{K}{r^2} \cos\theta \tag{8}$$

One of the assumptions made in the theory is that the fluid particles have no radial velocity at the surface of the cylinder where $r = r_o$; consequently, by setting equation (8) equal to zero the value of K is obtained as

$$K = V_o \, r_o^2$$

similarly the speed along the tangent to the section is:

$$V_s = \frac{\partial \Phi}{r \, \partial \theta} = \left[V_o \sin\theta \left(1 + \frac{r_o^2}{r^2}\right) - \frac{K'}{\pi r}\right]_{r=r_o} \tag{9}$$

$$= 2 \, V_o \sin\theta + V_1$$

in which V_1 represents $\frac{K'}{\pi r_o}$, and corresponds to the tangential speed component of the circulatory motion at the surface of the cylinder.

The velocity field resulting from the superposition of the two members of equation (9) gives the diagram of tangential velocity of the air on the cylinder surface shown in Fig. 4. In this particular case the values of V_o and V_1 chosen are those corresponding to figures given on photographs of Fig. 8; V_s is plotted radially instead of tangentially for pictorial purposes, positive outside the section and negative inside. Due to the sine term of the above equation the two terms are added in the first and second quadrant and subtracted in the third and fourth. The resulting figure is symmetric about the Z axis, with a much larger resultant velocity on the upper semicircle than on the lower.

If Bernoulli's equation is applied to this steady flow one finds that the pressure will be greater where the velocity is less, and vice versa; hence a force is produced along the Z axis, at right angles to the X axis, directed from the side of lesser velocity towards that of greater velocity.

The theoretical magnitude of this lateral force acting on a surface element $ds = r_o \, d\theta$, along the whole length L, of the cylinder, by Bernoulli's equation is:

$$\Delta p \, (L \, r_o \, d\theta) = \frac{W}{2g} (V_o^2 - V^2) \quad (L \, r_o \, d\theta) \qquad (10)$$

which, on substituting V_s for V (equation 9) gives:

$$\Delta p(Lr_o d\theta) = \frac{W}{2g}(Lr_o)\ V_o^2 - 4V_o^2 \sin^2\theta - 4V_oV_1 \sin\theta - V_1^2)\ d\theta \qquad (11)$$

Due to the symmetry of the flow about the Z axis the total force is obtained by integrating the infinitesimal projections on this axis between the limits $\pi/2$ and $-\pi/2$, and is therefore:

$$F_Z = -\int_{-\pi/2}^{\pi/2} (P - P_o)\ \ (L\ r_o)\ \sin\theta\ d\theta$$

$$= -\frac{W}{2g}(Lr_o)\int_{-\pi/2}^{\pi/2}(V_o^2 - 4\ V_o^2\ \ \sin^2\theta - 4\ V_oV_1\ \ \sin\theta - V_1^2)\sin\theta\ d\theta$$

simplifying to $F_Z = -\frac{W}{2g}\ L\ (2\ \pi\ r_o\ V_1)\ V_o = -\frac{W}{g}\ L\ \Gamma\ V_o$ (12)

The resultant force in the direction of the X axis is obtained likewise, as:

$$F_X = -\int_{-\pi/2}^{\pi/2} (P - P_o)\ L\ r_o\ \cos\theta\ d\theta = 0 \qquad (13)$$

EXPERIMENTAL RESULTS ON MODEL ROTATING CYLINDERS.

Besides Dr. Wolff's experiments, which apply only to airfoils, and some carried out at the Göttingen Laboratory, the most complete and recent set of experiments available on the subject is that of the N.A.C.A. Technical Note No. 209. These tests on rotating cylinders of various cross sections were made at the Langley Memorial Aeronautical Laboratory to determine the air force acting on rotating cylinders set transversely to the air flow. The cylinders were tested in infinite length-diameter ratio, by letting the length extend over the whole diameter of the tunnel section, thus very closely simulating a two-dimensional air flow. The remainder of the program consisted in recording the drag, the cross wind force, and the power required to rotate the cylinders for the controlled combination of this translational and circulatory motion.

The experimental data of Table I, as well as Figs. 5, 6, and 8, all referring to the circular cylinder, are taken from the above reference.

Following are the most relevant points of these tests on rotating cylinders, as presented in Technical Note No. 209.

(a) The cross wind force does not appear before a ratio r, (peripheral velocity/translational velocity) of 0.5 is reached* (Fig. 6).

(b) Between the ratio $r = .5$ and $r = 2$ the cross wind force increases steadily through quite a range with no appreciable variation in the drag (Fig. 5).

(c) The maximum value of L/D of 7.8 is obtained when $r = 2.5$ approximately (Fig. 5).

(d) The drag along the axis of the tunnel (for $V_0 = 15$ m/s) varies from a maximum at $V_1 = 0$ to a minimum for the range of $r = 0.5$ to $r = 2.0$, from which point it increases again and at the abscissa of greatest L/D the drag coefficient becomes almost identical with that of the stationary cylinder (see Figs. 5 and 6).

* This is particularly true for the 15 m/s airspeed.

In order to directly compare the experimental cross wind forces determined from wind tunnel tests and the theoretical ones computed according to equation (12), the latter have been tabulated also in Table I. From these data, the dotted line of Fig. 6, representing theoretical cross wind force coefficients, is derived for the same range of speeds and in the same manner as those of the experimental curve. Fig. 7 is derived from Fig. 6 and gives the ratio of experimental to theoretical C.W.F. coefficients against the values of V_1/V_0.

As noted in Fig. 6, there is a considerable discrepancy between experimental and theoretical coefficient curves which was not wholly unexpected, considering the assumptions involved in the theory. In addition to this quantitative discrepancy an interesting point to be noticed is the immediate appearance of a finite theoretical C.W.F., while the experimental does not start until a ratio V_1/V_0 of .5 is reached. These discrepancies taken together point out the importance of using discretion in the application of theoretical cross wind forces to practical problems.

THE FLETTNER ROTOR SHIP "BUCKAU."

In the previous section the discrepancies existing between theoretical and experimental cross wind forces on rotating cylinders have been indicated. In the following paragraphs the speed of the Flettner rotor ship, "Buckau," is computed by applying both propulsive forces, using experimental and theoretical coefficients, so that we may be able to see which of the two gives closer results to the actual performance.

Accounts of the "Buckau" have appeared in various publications and need no further comment here. In brief, the inventor introduces a new method of marine propulsion by bringing into effect the force delivered by the wind working on two vertical cylinders rotating in the longitudinal plane of symmetry of the vessel. The main object of using two cylinders is obviously for maneuvering purposes, the proper balancing of the components being accomplished by varying the ratio of the peripheral speed of each cylinder to that of the prevailing wind.

From what information it is possible to obtain, the "Buckau" has a displacement of about 680 tons, and two cylinders 18.5 m (60.7 ft.) high and 2.8 m (9.2 ft.) in diameter which are revolved at the rate of 120 R.P.M. by two 11 kilowatt, 220 volt motors, these being in turn operated by a 45 HP. Diesel engine. Under ordinary winds, it has been reported that it makes five or six knots.

If the vessel maneuvers in a wind of 5 m/s, which corresponds to an ordinary wind of 9.7 knots, the theoretical thrust normal to the wind direction for each cylinder will be:

$$F_z = \frac{W}{g} L\,(2\,\pi\,r_o\,V_1)\,V_o = 3175 \text{ lb. } (1440 \text{ kg})$$

and for both cylinders the thrust would therefore be 6350 lb. (2880 kg).

The indicated HP. of an engine necessary to propel such a vessel as the "Buckau" at a speed of 12 knots would be (Ref. 12):

$$HP. = \frac{1}{160} D^{2/3} V^3 = \frac{1}{160} (680)^{2/3} (12)^3 = 835.$$

Assuming a propulsive coefficient of .55, the above horsepower is equivalent to an effective propulsion of 12,450 lb. The speed corresponding to the above propulsive force is therefore:

$$V = 12 \left(\frac{6350}{12450}\right)^{1/2} = 8.60 \text{ knots,}$$

which is considerably better than the predicted performance of 5 or 6 knots.

On the other hand, by using wind tunnel data for the same condition, namely, 5 m/s and 120 R.P.M., we get:

$$F_z = 2\ C_C\ Aq$$

$$= 2\ (8.59)\ (51.7)\ 1.535 = 1368 \text{ kg}$$

$$= 3016 \text{ lb.}$$

where C_C corresponding to $r = 3.45$ is taken from Fig. 6, A is the projected area of each cylinder on the "Buckau" and q is the aerodynamic pressure $\frac{1}{2} \rho V_0^2$. The speed corresponding to this propulsive force is

$$V = 12 \left(\frac{3010}{12450}\right)^{1/2} = 5.90 \text{ knots.}$$

Ship speeds similarly computed for the wind speeds therein indicated and a constant cylinder rotational speed of 120 R.P.M. are given for comparison in Table II.

The comparison speaks for itself and points out at once the departure between corresponding speeds computed by the two sets of data. One feature of these figures is that increasing the wind speed from 5 to 15 m/s (by 200 per cent), the ship speed increases nearly 74 per cent when employing theoretical thrust and only 20 per cent when experimental data are used.

GENERAL DISCUSSION.

There are many phases of the rotor question which are extremely interesting but which require too extensive treatment to be included in this paper. However, it does seem appropriate at this time to briefly discuss the most interesting points relating to the theory of these rotating cylinders and their effect upon the characteristics of the ship.

The fact that a much closer agreement is obtained between theoretical and experimental cross wind forces on wing sections developed by this theory (Reference 13) than in the apparently simpler case of these rotating cylinders is undoubtedly due to the less problematic conditions of the flow around such wing sections (Reference 14). The main difficulty met in the theory, as pointed out by eminent authorities, lies in the determination of the actual circulation and relative distribution around the body; for, while in the case of rotating cylinders the circulation is assumed to be inversely as the distance r from the center, unimpaired by viscosity, slip, and such detrimental elements, in the wing theory advocated by the Göttingen school this circulation is actually determined from the condition of the flow itself.

Were the air particles free from attraction and the motion frictionless, as assumed in the theory, there would be, in this case, no circulation at all, and no Magnus effect experienced by the cylinder, since the air next to it would not be affected by its rotation. This is contrary to common experience since the stratum of air in the immediate vicinity of the cylinder surface adheres to the latter, some circulation resulting therefrom, even though quantitatively less than the assumed value, and dissipated at a very short distance from the cylinder surface (Reference 15).

It goes without saying that the drag in the direction of the wind is of negligible effect in the case of the rotor ship when it is compared to the lateral wind force which would be felt by the vessel under the action of a sail of the right proportions. This drag entirely unaccounted for in the Kutta-Joukowski theory (equation 13) in the case of the model cylinder can easily be accounted for, at least qualitatively by such a method as Karman has employed in the theory of vortex motion. The "Buckau," fitted with rotating cylinders, will be therefore at an advantage in sudden squalls since the heeling couple caused by the transverse component of wind force (drag of rotating cylinder) is considerably less than that caused, were it fitted with sails, even though the point of application of this lateral force remains practically at the same height above the center of lateral resistance of the vessel in both cases.

From the propulsion point of view, this dynamic drag in the direction of the wind is not an entire detriment, for it can at times be utilized by sailing whenever possible along the resultant of the two existing forces, the effective propulsive effort along any other course is (C.W.F.) $\sin\gamma \pm$ (drag) $\cos\gamma$, γ being the acute angle between the wind vector and the normal to the ship course; the minus sign for head winds, and the plus sign for lee winds. Table I shows the gain of propulsive force that can be realized by sailing along the direction of the resultant.

Another favorable element contributed by converting the "Buckau" from sail to rotor ship is the lowering of the center of gravity of the vessel caused by the difference of the weights of the two riggings and the corresponding distances of the c.g. above deck. Thus assuming the old and new rigging to weigh 35 and 8 tons respectively, and the c.g. 14 ft. and 8 ft. respectively above deck, a lowering of the ship's c.g. results which amounts to approximately 1/2 foot. It is obvious therefore, that from a seaworthiness point of view the "Buckau" has a greater reserve of dynamical stability in this case than were it fitted with sails.

Aside from the above considerations, it may be questioned how closely the Flettner rotors provided with top disks simulate the conditions existing in the infinite length model. The smoke photographs of Fig. 8, taken for the conditions indicated thereon, surely give some light on this question. These actual pictures besides showing a departure on the wake side of the actual flow from the theoretical two-dimensional one of Fig. 3, also indicate to what extent the stratum of air in the immediate vicinity of the cylinder is being affected, giving thus a rough idea of the diameter of end disk necessary to prevent the air stream from spilling over the end.

There is no object in going into finer details as the work so far done on rotating cylinders is only of a preliminary nature, and more extensive work is expected to be carried out in the near future. The published accounts of the rotor ship and the above speed calculations leave very little doubt that Flettner's expectations have been materialized and that the ship can be considered as giving a very good auxiliary means of propulsion.

CONCLUSIONS.

The results just discussed indicate that in spite of the fact that the Kutta-Joukowski theory is based on the motion of perfect fluids, there is nevertheless a certain correspondence between theoretical and experimental cross wind forces of rotating cylinders.

This analysis shows that it is unwise to use theoretical cross wind force. Wind tunnel data give results which are much closer to the actual performance of the "Buckau."

In conclusion, it may be safely stated that in spite of the encouraging results obtained in these preliminary investigations on rotating cylinders, considerable research still remains to be done in this new field of aerodynamics if it is expected that the latter will find an application in the auxiliary propulsion of ships as well as in aeronautics.

TABLE I.

Experimental and theoretical cross wind forces on a revolving 4.5 inch diameter circular cylinder

V_o m/s	R.P.M.	V_1 m/s	r V_1/V_o	Experimental Drag kg	C.W.F. kg	Result kg	C_D	C_C
15	25	.148	.010	1.136	-.010	1.136	.925	-.008
15	500	2.96	.200	1.136	+.010	1.136	.925	-.008
15	900	5.33	.360	1.26	-.020	1.026	.835	-.016
15	1020	6.05	.408	.942	-.022	.942	.766	-.018
15	1115	6.82	.460	,852	.007	.852	.693	-.006
15	1240	7.35	.496	.777	+.003	.777	.632	+.002
15	1300	7.70	.520	.754	.018	.754	.614	+.014
15	1400	8.29	.560	.740	.150	.755	.602	+.122
15	1500	8.88	.600	.744	.283	.795	.605	.230
15	1600	9.48	.640	.744	.400	.845	.605	.326
15	1700	10.07	.680	.750	.598	.962	.610	.487
15	1780	10.55	.712	.751	.662	.998	.611	.537
15	1800	10.65	.720	.754	.673	1.011	.614	.548
15	1900	11.25	.760	.757	.798	1.099	.616	.650
15	2000	11.86	.800	.759	.873	1.157	.618	.710
15	2080	12.32	.832	.765	.868	1.157	.622	.706
15	2100	12.45	.840	.764	.997	1.256	.622	.811
15	2200	13.04	.880	.764	1.073	1.319	.622	.873
15	2230	13.14	.888	.787	1.158	1.400	.640	.942
15	2300	13.61	.920	.772	1.188	1.416	.628	.967
15	2400	14.21	.968	.754	1.278	1.480	.614	1.040
15	2500	14.79	1.000	.742	1.338	1.529	.604	1.089
15	2600	15.39	1.040	.729	1.468	1.636	.593	1.194
15	2620	15.50	1.048	.724	1.303	1.492	.589	1.060
15	2700	15.98	1.080	.710	1.578	1.726	.578	1.284

$C_D = \frac{D}{qs}$ $C_C = \frac{C.W.F.}{qs}$ $S = 0.1741\ m^2$ $q = \frac{1}{2}\rho V^2\ kg/m^2 =$

$13.81\ kg/m^2$

Table 1 (Cont'd)

Experimental and theoretical cross wind forces on a revolving 4.5 inch diameter circular cylinder.

V_0 m/s	R.P.M.	V_1 m/s	r V_1/V_0	Experimental Drag kg	C.W.F. kg	Result kg	C_D	C_C
10	1300	7.70	.780	.353	.308	.468	.646	.563
10	1500	8.88	.900	.351	.418	.545	.642	.764
10	1700	10.06	1.020	.338	.636	.719	.618	1.163
10	1900	11.23	1.140	.331	.758	.828	.605	1.386
10	2000	12.42	1.260	.322	.978	1.027	.589	1.789
10	2300	13.60	1.380	.324	1.083	1.130	.593	1.980
10	2500	14.79	1.500	.332	1.293	1.336	.607	2.362
10	2700	15.98	1.620	.334	1.403	1.443	.611	2.564
10	2900	17.15	1.740	.346	1.443	1.486	.633	2.639
$q = 6.15$ kg/m²								
7	1800	10.65	1.540	.167	.660	.680	.624	2.46
7	2100	12.45	1.790	.173	.860	.877	.646	3.21
7	2400	14.21	2.050	.181	1.140	1.154	.676	4.26
7	2700	15.98	2.300	.197	1.365	1.383	.736	5.10
7	3000	17.76	2.560	.222	1.700	1.714	.829	6.35
7	3300	19.55	2.820	.256	1.945	1.964	.956	7.26
7	3600	21.32	3.070	.287	2.210	2.232	1.070	8.25
$q = 3.01$ kg/m²								
5	1800	10.65	2.16	.085	.605	.613	.622	4.43
5	2100	12.45	2.51	.105	.820	.828	.769	6.00
5	2400	14.21	2.87	.130	.995	1.005	.952	7.28
5	2700	15.98	3.23	.151	1.110	1.119	1.105	8.13
5	3000	17.76	3.59	.168	1.170	1.180	1.230	8.57
5	3300	19.55	3.95	.188	1.250	1.263	1.376	9.15
5	3600	21.32	4.32	.196	1.295	1.311	1.434	9.48
$q = 1.535$ kg/m²								

TABLE I (Cont'd

Experimental and theoretical cross wind forces on a revolving 4.5 inch diameter circular cylinder.

V_o m/s	R.P.M.	Theoretical C.W.F. kg	Theoretical C_c	Ratio Exp. C.W.F. / Th. C.W.F.
15	25	.15	0.06	
15	500	2.95	1.22	
15	900	5.31	2.20	
15	1020	6.03	2.50	
15	1115	6.79	2.82	
15	1240	6.33	2.62	
15	1300	7.67	3.18	.004
15	1400	8.25	3.42	.035
15	1500	8.85	3.68	.063
15	1600	9.45	3.92	.083
15	1700	10.02	4.21	.116
15	1780	10.51	4.36	.123
15	1800	10.60	4.40	.125
15	1900	11.21	4.65	.140
15	2000	11.82	4.91	.145
15	2080	12.28	5.10	.138
15	2100	12.40	5.15	.157
15	2200	13.00	5.39	.162
15	2220	13.10	5.44	.173
15	2300	13.57	5.63	.172
15	2400	14.16	5.87	.178
15	2500	14.73	6.11	.179
15	2600	15.32	6.36	.188
15	2620	15.44	6.41	.166
15	2700	15.93	6.61	.195

TABLE I (Cont'd).

Experimental and theoretical cross wind forces on a revolving 4.5 inch diameter circular cylinder.

V_o m/s	R.P.M.	Theoretical C.W.F. kg	Theoretical C_c	Ratio Exp. C.W.F. / Th. C.W.F.
10	1300	5.12	4.78	.118
10	1500	5.90	5.50	.139
10	1700	6.68	6.24	.187
10	1900	7.47	6.97	.199
10	2000	8.26	7.71	.232
10	2300	9.04	8.44	.235
10	2500	9.83	9.18	.257
10	2700	10.62	9.90	.259
10	2900	11.40	10.64	.248
7	1800	4.96	9.45	.261
7	2100	5.79	11.01	.292
7	2400	6.62	12.60	.338
7	2700	7.44	14.18	.360
7	3000	8.27	15.76	.402
7	3300	9.10	17.31	.420
7	3600	9.93	18.91	.437
5	1800	3.54	13.26	.334
5	2100	4.14	15.50	.387
5	2400	4.73	17.72	.412
5	2700	5.31	19.88	.408
5	3000	5.90	22.10	.388
5	3300	6.50	24.35	.376
5	3600	7.08	26.50	.358

Table II.

Ship Speed by Theoretical and Experimental C.W.F.

Wind speed in m/s	15	10	7	5
" " " knots	29.2	19.5	13.6	9.7
Speed of "Buckau" by theory (knots)	11.70	9.55	7.98	6.74
Speed by wind tunnel data (knots)	7.10	6.95	6.90	5.90

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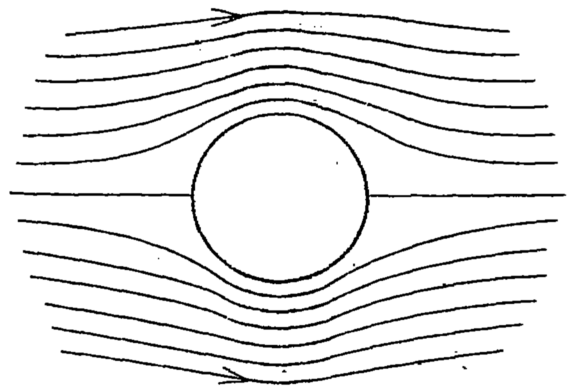

Fig.1 Uniplanar uniform flow around circular cylinder.

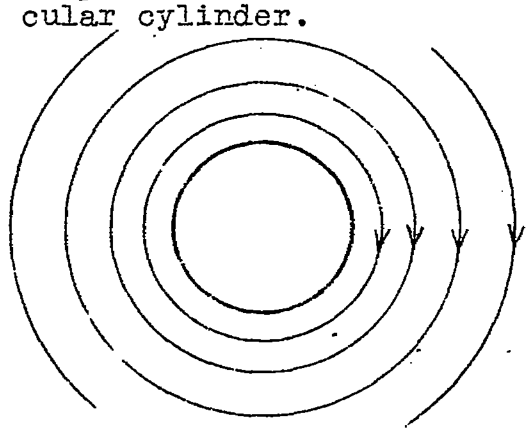

Fig.2 Uniplanar flow around circular cylinder considered as a columnar vortex of strength Γ.

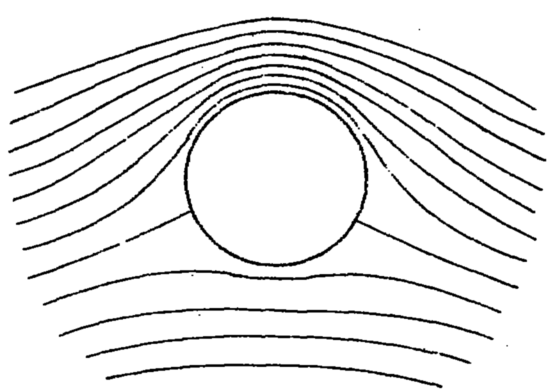

Fig.3 Superposition of two preceding flows.

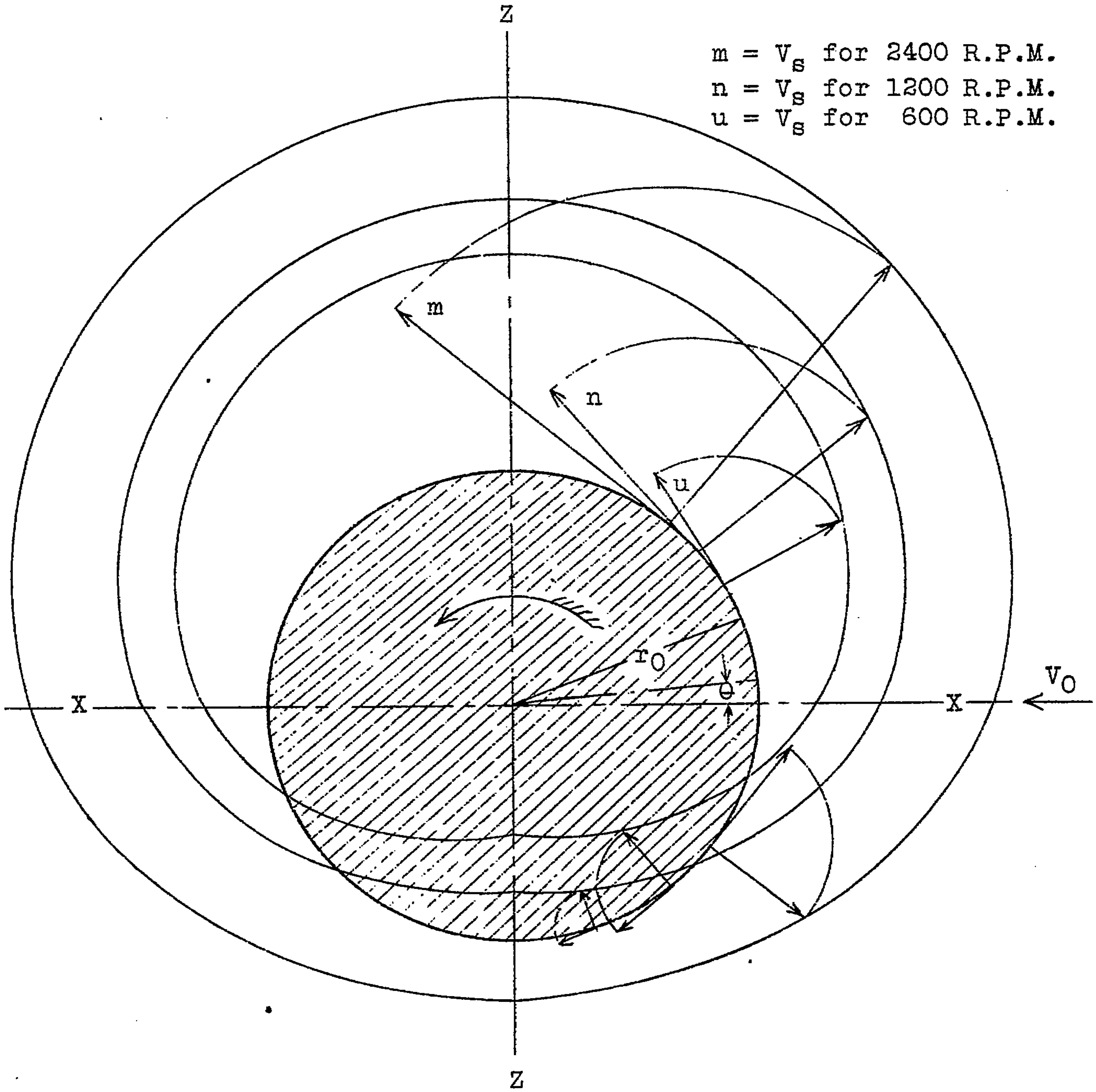

Fig.4 V_s Distribution.

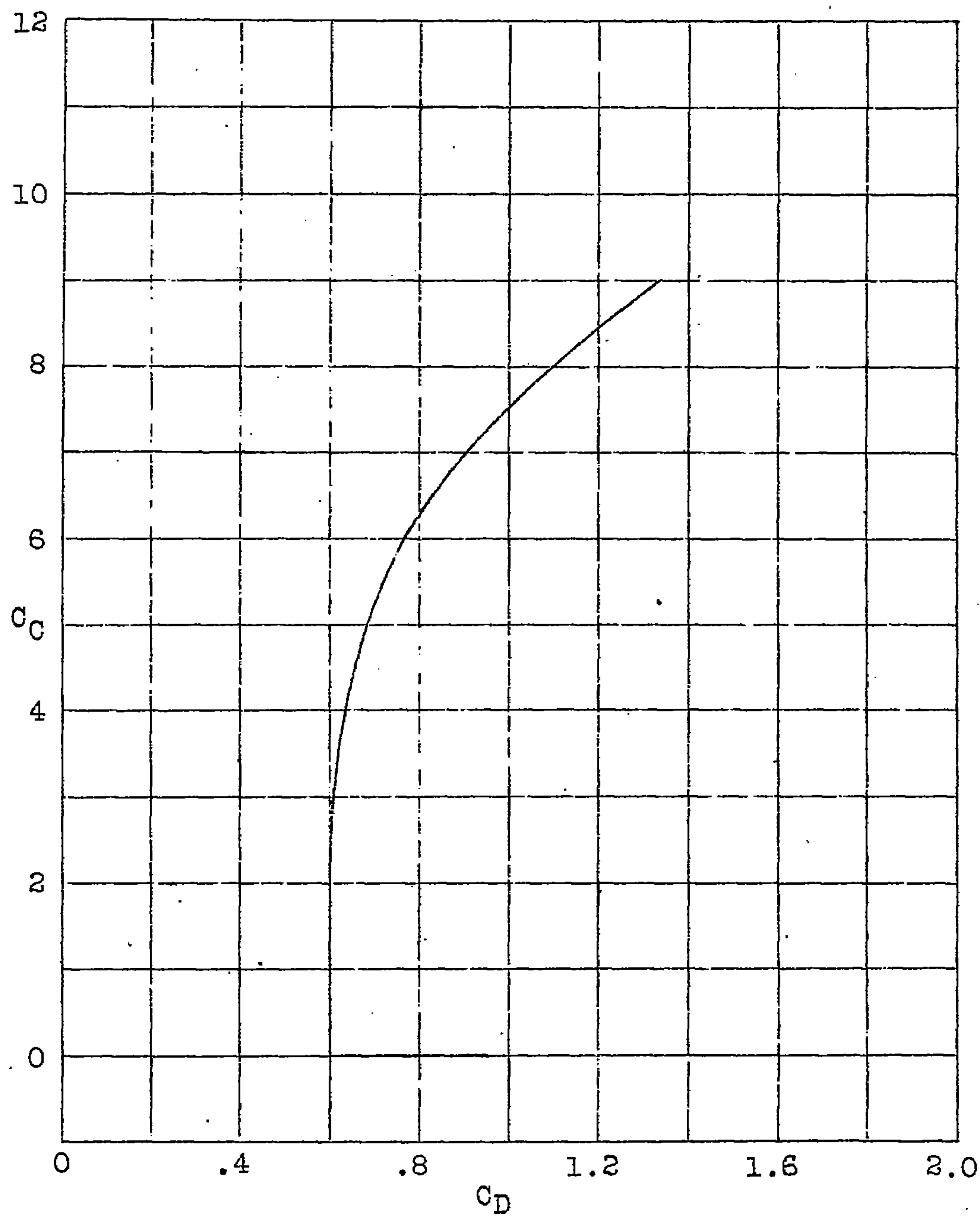

Fig.5 Cross wind & drag coeffs. for rotating circular cylinder.

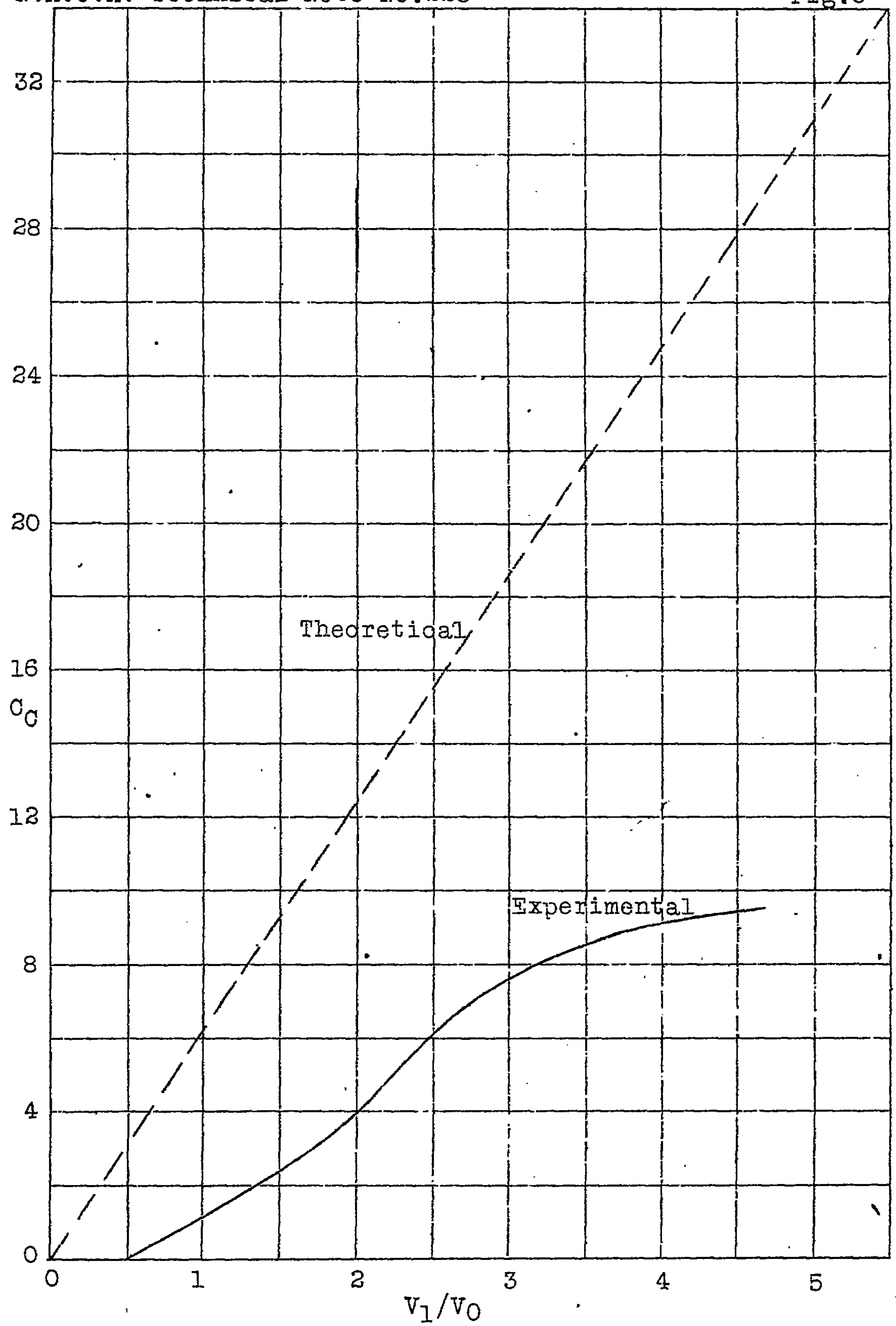

Fig.6 Cross wind force coeff. vs. V_1/V_0 for rotating cylinder.

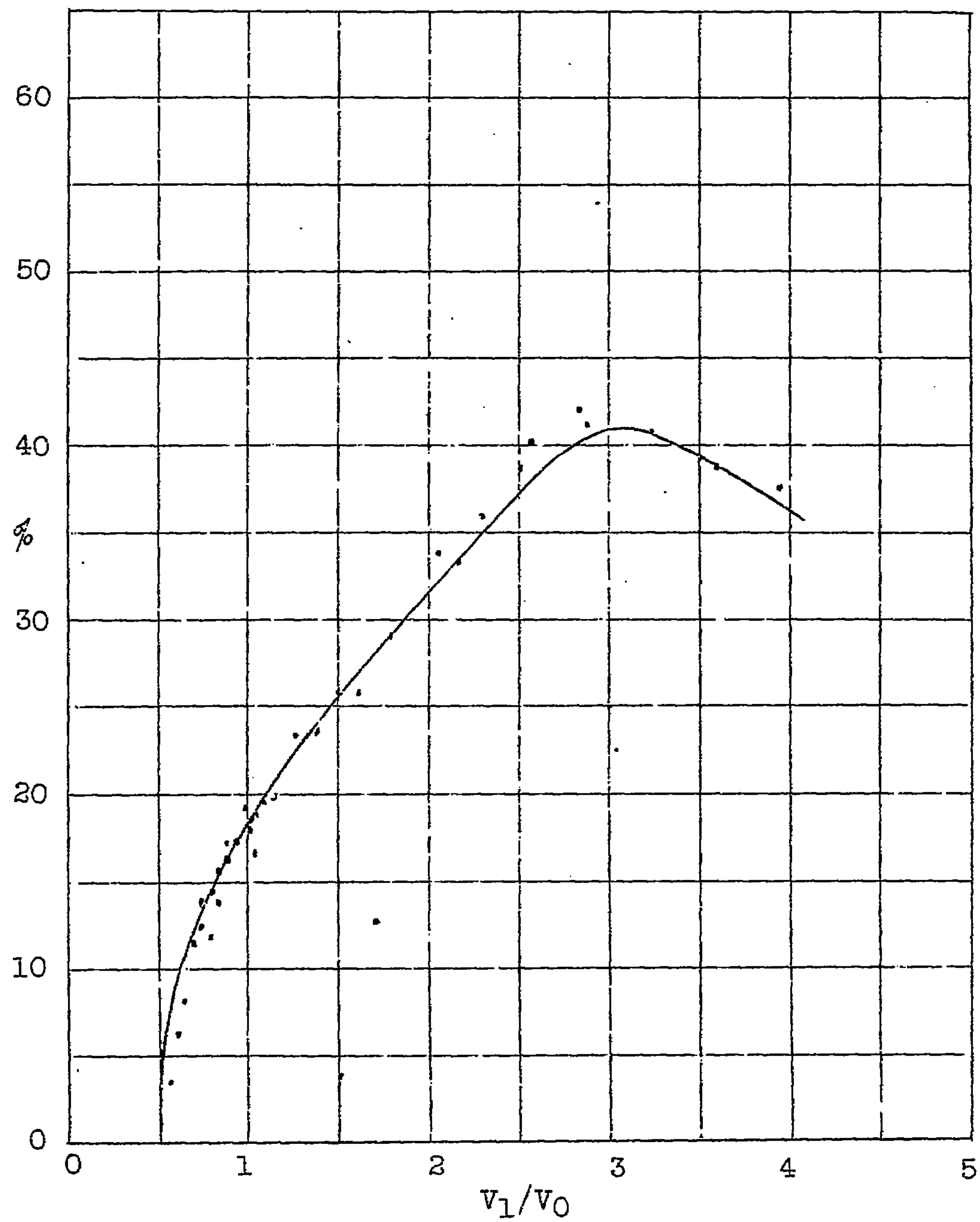

Fig.7 Ratio experimental C_c to theoretical C_c vs V_1/V_0.

Fig.8 Photographs of smokestream around 4 1/2" revolving cylinder.

5945 A.S.